OCR
Oxford Cambridge and RSA

A Level Mathematics A

H240/01 Pure Mathematics

Wednesday 6 June 2018 – Morning

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

Model Solutions

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INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

1 The points A and B have coordinates (1, 5) and (4, 17) respectively. Find the equation of the straight line which passes through the point (2, 8) and is perpendicular to AB. Give your answer in the form $ax+by=c$, where a, b and c are constants. **[4]**

1 gradient of line AB $= \frac{17-5}{4-1} = \frac{12}{3} = 4$

gradient of line perpendicular to AB $= -\frac{1}{4} = m$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{1}{4}(x - 2)$

$4y - 32 = -x + 2$

$4y = 34 - x$

$4y + x = 34$

2 **(i)** Use the trapezium rule, with four strips each of width 0.5, to estimate the value of

$$\int_0^2 e^{x^2}\,dx$$

giving your answer correct to 3 significant figures. **[3]**

2 i) $\int_0^2 e^{x^2}\,dx \approx \frac{0.5}{2}\left(e^0 + e^{2^2} + 2\left(e^{0.5^2} + e^{1^2} + e^{1.5^2}\right)\right)$

$= 20.6446$

$= 20.6$

(ii) Explain how the trapezium rule could be used to obtain a more accurate estimate. **[1]**

ii Use more strips / trapezia over the interval

3 In this question you must show detailed reasoning.

Find the two real roots of the equation $x^4 - 5 = 4x^2$. Give the roots in an exact form. **[4]**

3. $x^4 - 5 = 4x^2$

$x^4 - 4x^2 - 5 = 0$

$(x^2 + 1)(x^2 - 5) = 0$

$x^2 + 1 = 0$ gives no real solutions

So $x^2 = 5$

$x = \pm\sqrt{5}$

4 Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n. **[4]**

4. Consider the two cases, if n is even and if n is odd

If n is even, let $n = 2m$

$n^3 + 3n - 1 = (2m)^3 + 3(2m) - 1$

$= 8m^3 + 6m - 1$

$= 2(4m^3 + 3m) - 1$

$2(4m^3+3m) - 1$ is odd, hence $n^3 + 3n - 1$ is odd when n is even. Any number multiplied by an even number is even ∴ minusing 1 leads to an odd number.

If n is odd, let $n = 2m + 1$

$n^3 + 3n - 1 = (2m+1)^3 + 3(2m+1) - 1$

$= (2m+1)(4m^2 + 4m + 1) + 6m + 3 - 1$

$= 8m^3 + 8m^2 + 2m + 4m^2 + 4m + 1 + 6m + 2$

$= 8m^3 + 12m^2 + 12m + 3$

$= 2(\cancel{8}4m^3 + 6m^2 + 6m + 1) + 1$

$2(4m^3 + 6m^2 + 6m + 1) - 1$ is odd, hence

$n^3 + 3n - 1$ is odd when n is odd

because any number multiplied by 2 is even ∴ minusing 1 makes it odd

∴ $n^3 + 3n - 1$ is odd for all integers n

5 The equation of a circle is $x^2 + y^2 + 6x - 2y - 10 = 0$.

(i) Find the centre and radius of the circle. **[3]**

5 i) $x^2 + y^2 + 6x - 2y - 10 = 0$

$(x+3)^2 - 9 + (y-1)^2 - 1 - 10 = 0$

$(x+3)^2 + (y-1)^2 = 20$

Centre : $(-3, 1)$

Radius : $\sqrt{20}$

(ii) Find the coordinates of any points where the line $y = 2x - 3$ meets the circle $x^2 + y^2 + 6x - 2y - 10 = 0$. **[4]**

ii) Sub $y = 2x - 3$ into $x^2 + y^2 + 6x - 2y - 10 = 0$

$x^2 + (2x-3)^2 + 6x - 2(2x-3) - 10 = 0$

$x^2 + 4x^2 - 12x + 9 + 6x - 4x + 6 - 10 = 0$

$5x^2 - 10x + 5 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$, $y = 2 - 3 = -1$

∴ Coordinates are $(1, -1)$

(iii) State what can be deduced from the answer to part **(ii)** about the line $y = 2x - 3$ and the circle $x^2 + y^2 + 6x - 2y - 10 = 0$. **[1]**

iii) The line is tangent to the circle because it only crosses once

Alternativeuy, discriminant gives 0 which means it's only a tangent.

6 The cubic polynomial $f(x)$ is defined by $f(x) = 2x^3 - 7x^2 + 2x + 3$.

(i) Given that $(x - 3)$ is a factor of $f(x)$, express $f(x)$ in a fully factorised form. **[3]**

6 i)

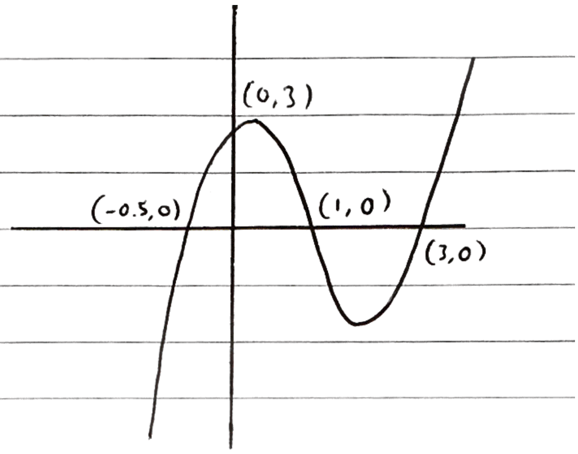
$$\begin{array}{r} 2x^2 - x - 1 \\ x - 3 \,\big)\, \overline{2x^3 - 7x^2 + 2x + 3} \\ \underline{2x^3 - 6x^2} \qquad\qquad \\ -x^2 + 2x + 3 \\ \underline{-x^2 + 3x} \quad\;\; \\ -x + 3 \\ \underline{-x + 3} \\ 0 \end{array}$$

$2x^3 - 7x^2 + 2x + 3 = (x - 3)(2x^2 - x - 1)$

$= (x - 3)(2x + 1)(x - 1)$

(ii) Sketch the graph of $y = f(x)$, indicating the coordinates of any points of intersection with the axes. **[2]**

ii



(iii) Solve the inequality $f(x) < 0$, giving your answer in set notation. **[2]**

iii The parts of the graph which are below the x axis are

$x < -0.5$ and $1 < x < 3$

$\{x : x < -0.5\} \cup \{x : 1 < x < 3\}$

(iv) The graph of $y = f(x)$ is transformed by a stretch parallel to the x-axis, scale factor $\frac{1}{2}$. Find the equation of the transformed graph. **[2]**

iv. Transformed graph is $f(2x)$

$f(2x) = 2(2x)^3 - 7(2x)^2 + 2(2x) + 3$

$= 16x^3 - 28x^2 + 4x + 3$

7 Chris runs half marathons, and is following a training programme to improve his times. His time for his first half marathon is 150 minutes. His time for his second half marathon is 147 minutes. Chris believes that his times can be modelled by a geometric progression.

(i) Chris sets himself a target of completing a half marathon in less than 120 minutes. Show that this model predicts that Chris will achieve his target on his thirteenth half marathon. **[4]**

7 i) $r = \frac{147}{150} = 0.98$

$a = 150$

$u_n = 150(0.98)^{n-1}$

If $n = 12$, $u_{12} = 150 \times 0.98^{11} = 120.1$

If $n = 13$, $u_{13} = 150 \times 0.98^{12} = 117.7$

$u_{12} > 120$, $u_{13} < 120$ so 120 minutes will be achieved on the 13^{th} marathon

(ii) After twelve months Chris has spent a total of 2974 minutes, to the nearest minute, running half marathons. Use this model to find how many half marathons he has run. **[3]**

ii $S_n = \dfrac{a(1-r^n)}{1-r}$

$2974 = \dfrac{150(1-0.98^n)}{1-0.98}$

$59.48 = 150(1-0.98^n)$

$1-0.98^n = 0.3965$

$0.98^n = 0.6035$

$\log 0.98^n = \log 0.6035$

$n \log 0.98 = \log 0.6035$

$n = \dfrac{\log 0.6035}{\log 0.98}$

$n = 24.997$

$n = 25$

$\therefore$ He has run 25 half marathons

(iii) Give two reasons why this model may not be appropriate when predicting the time for a half marathon. **[2]**

iii he will not continue to improve forever.
variations in conditions will mean there is more variation in the times than the model suggests

8 **(i)** Find the first three terms in the expansion of $(4-x)^{-\frac{1}{2}}$ in ascending powers of x. **[4]**

8 i) $(4-x)^{-\frac{1}{2}} = \left[4(1-\frac{1}{4}x)\right]^{-\frac{1}{2}}$

$= \frac{1}{2}(1-\frac{1}{4}x)^{-\frac{1}{2}}$

$\frac{1}{2}(1-\frac{1}{4}x)^{-\frac{1}{2}} = \frac{1}{2}\left[1+(-\frac{1}{2})(-\frac{1}{4}x)+(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{1}{4}x)^2\right]$

$= \frac{1}{2}\left[1+\frac{1}{8}x+\frac{3}{128}x^2\right]$

$= \frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$

(ii) The expansion of $\dfrac{a+bx}{\sqrt{4-x}}$ is $16-x$ … . Find the values of the constants a and b. **[3]**

ii) $\dfrac{a+bx}{\sqrt{4-x}} = (a+bx)(4-x)^{-\frac{1}{2}}$

$= (a+bx)(\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2)$

$= \frac{1}{2}a+\frac{1}{16}ax+\frac{1}{2}bx+\dots$

$= \frac{1}{2}a + x(\frac{1}{16}a+\frac{1}{2}b)$

$\frac{1}{2}a = 16$

$a = 32$

$\frac{1}{16}a + \frac{1}{2}b = -1$

$\frac{32}{16} + \frac{1}{2}b = -1$

$2 + \frac{1}{2}b = -1$

$\frac{1}{2}b = -3$

$b = -6$

9 The function f is defined for all real values of x as $f(x) = c + 8x - x^2$, where c is a constant.

(i) Given that the range of f is $f(x) \leqslant 19$, find the value of c. **[3]**

9 i) Complete the square:

$c + 8x - x^2 = -(x-4)^2 + 16 + c$

$16 + c = 19$

$c = 3$

(ii) Given instead that $ff(2) = 8$, find the possible values of c. **[4]**

ii. $f(2) = c + 8(2) - 2^2$

$= c + 12$

$f(f(2)) = f(c+12)$

$8 = c + 8(c+12) - (c+12)^2$

$8 = c + 8c + 96 - c^2 - 24c - 144$

$8 = -c^2 - 15c - 48$

$c^2 + 15c + 56 = 0$

$(c+7)(c+8) = 0$

$c = -7$ or $c = -8$

10 A curve has parametric equations $x = t + \frac{2}{t}$ and $y = t - \frac{2}{t}$, for $t \neq 0$.

(i) Find $\frac{dy}{dx}$ in terms of t, giving your answer in its simplest form. **[4]**

10 i) $x = t + \frac{2}{t}$ $\quad y = t - \frac{2}{t}$

$\frac{dx}{dt} = 1 - \frac{2}{t^2}$ $\quad \frac{dy}{dt} = 1 + \frac{2}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1 + \frac{2}{t^2}}{1 - \frac{2}{t^2}}$ $= \frac{t^2 + 2}{t^2 - 2}$

(ii) Explain why the curve has no stationary points. **[2]**

ii Stationary points occur when $\frac{dy}{dx} = 0$

$0 = \frac{t^2+2}{t^2-2}$

$0 = t^2 + 2$

$t^2 = -2$

$t^2 = -2$ has no solutions, hence there are no stationary points

(iii) By considering $x + y$, or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets. **[4]**

iii. $x + y = t + \frac{2}{t} + t - \frac{2}{t} = 2t$

$x = t + \frac{2}{t}$

$x = \frac{1}{2}(x+y) + \frac{2}{\frac{1}{2}(x+y)}$

$x(x+y) = \frac{1}{2}(x+y)^2 + 4$

$x^2 + xy = \frac{1}{2}(x^2 + 2xy + y^2) + 4$

$2x^2 + \cancel{2xy} = x^2 + \cancel{2xy} + y^2 + 8$

$x^2 - y^2 = 8$

11 In a science experiment a substance is decaying exponentially. Its mass, M grams, at time t minutes is given by $M = 300e^{-0.05t}$.

(i) Find the time taken for the mass to decrease to half of its original value. **[3]**

11 i) when $t = 0$, $M = 300$ so the original mass is 300g

We want to find when it reaches 150g

$150 = 300e^{-0.05t}$

$0.5 = e^{-0.05t}$

$\ln 0.5 = -0.05t$

$t = -20\ln 0.5$

$t = 13.8629...$

$t = 13.9$ minutes

A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

(ii) Find the time at which both substances are decaying at the same rate. **[8]**

ii) let $M_2 = ae^{bt}$

when $t = 0$, $M_2 = 400$

$400 = a$

when $t = 10$, $M_2 = 320$

$320 = 400e^{bt}$

$0.8 = e^{bt}$

$bt = \ln 0.8$

$10b = \ln 0.8$

$b = -0.0223$

$M_2 = 400e^{-0.0223t}$

We need to find out the rate of decay of both substances, so you need to differentiate both equations

$M_1 = 300e^{-0.05t}$ $\quad$ $M_2 = 400e^{-0.0223t}$

$\frac{dM_1}{dt} = -15e^{-0.05t}$ $\quad$ $\frac{dM_2}{dt} = -8.9257e^{-0.0223t}$

$-15e^{-0.05t} = -8.9257e^{-0.0223t}$

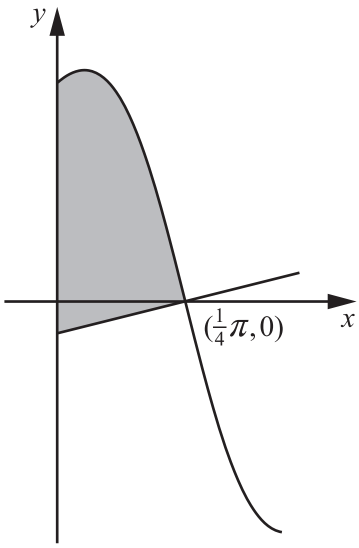
$1.6805 = e^{0.0277t}$

$\ln(1.6805) = 0.0277t$

$t = 18.74$

$t = 18.7$ minutes

12 In this question you must show detailed reasoning.



The diagram shows the curve $y = \dfrac{4\cos 2x}{3 - \sin 2x}$, for $x \geqslant 0$, and the normal to the curve at the point $(\frac{1}{4}\pi, 0)$. Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the y-axis is $\ln\dfrac{9}{4} + \dfrac{1}{128}\pi^2$. **[10]**

12 Split the area into the part between the curve and the x axis, and the triangle underneath.

Area of curve section $= \displaystyle\int_0^{\frac{\pi}{4}} \frac{4\cos 2x}{3 - \sin 2x}\, dx$

$= \Big[-2\ln|3 - \sin 2x| \Big]_0^{\frac{\pi}{4}}$

$= -2\ln|3 - \sin\frac{\pi}{2}| + 2\ln|3 - \sin 0|$

$= -2\ln|3 - 1| + 2\ln|3 - 0|$

$= 2\ln 3 - 2\ln 2$

$= 2\ln\frac{3}{2}$

$= \ln\frac{9}{4}$

For the triangle we need to find where it intercepts the y axis. For this we need to find the gradient of the line

$$\frac{dy}{dx} = \frac{(3 - \sin 2x)(-8\sin 2x) - (4\cos 2x)(-2\cos 2x)}{(3 - \sin 2x)^2}$$

$$\frac{dy}{dx} = \frac{-24\sin 2x + 8\sin^2 2x + 8\cos^2 2x}{(3 - \sin 2x)^2}$$

$$\frac{dy}{dx} = \frac{-24\sin 2x + 8}{(3 - \sin 2x)^2}$$

At $x = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{-24\sin\frac{\pi}{2} + 8}{(3 - \sin\frac{\pi}{2})^2}$

$$\frac{dy}{dx} = \frac{-24 + 8}{(3 - 1)^2}$$

$$\frac{dy}{dx} = \frac{-16}{4}$$

$$\frac{dy}{dx} = -4$$

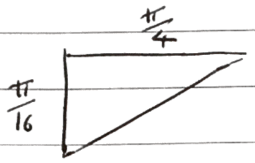
So the gradient of the normal to this point is $\frac{1}{4}$

Therefore the equation of the line is

$$y - 0 = \frac{1}{4}\left(x - \frac{\pi}{4}\right)$$

$$y = \frac{1}{4}x - \frac{\pi}{16}$$

when $x = 0$, $y = \frac{1}{16}\pi$



$$\text{Area} = \frac{1}{2} \times \frac{\pi}{16} \times \frac{\pi}{4} = \frac{\pi^2}{128}$$

$$\text{total area} = \ln\frac{9}{4} + \frac{\pi^2}{128}$$

13 A scientist is attempting to model the number of insects, N, present in a colony at time t weeks. When $t = 0$ there are 400 insects and when $t = 1$ there are 440 insects.

(i) A scientist assumes that the rate of increase of the number of insects is inversely proportional to the number of insects present at time t.

(a) Write down a differential equation to model this situation. **[1]**

13 i a) $\frac{dN}{dt} = \frac{k}{N}$

(b) Solve this differential equation to find N in terms of t. **[4]**

b) $\int N \, dN = \int k \, dt$

$$\frac{1}{2}N^2 = kt + c$$

when $t = 0$, $N = 400$:

$$\frac{1}{2}(400)^2 = c$$

$$c = 80000$$

when $t = 1$, $N = 440$

$$\frac{1}{2}(440)^2 = k + 80000$$

$$96800 = k + 80000$$

$$k = 16800$$

$\therefore \quad \frac{1}{2}N^2 = 16800t + 80000$

$N^2 = 33600t + 160000$

$N = \sqrt{33600t + 160000}$

(ii) In a revised model it is assumed that $\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$. Solve this differential equation to find N in terms of t. **[6]**

ii $\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$

$\int \frac{1}{N^2}\, dN = \int \frac{1}{3988} e^{-0.2t}$

$-N^{-1} = \frac{1}{3988} \cdot \frac{1}{-0.2} e^{-0.2t} + c$

$-N^{-1} = \frac{-5}{3988} e^{-0.2t} + c$

$\frac{-3988}{N} = -5e^{-0.2t} + k$

when $t = 0$, $N = 400$

$\frac{-3988}{400} = -5 + k$

$k = 5 - 9.97$

$k = -4.97$

$\frac{3988}{N} = 5e^{-0.2t} + 4.97$

$N = \frac{3988}{5e^{-0.2t} + 4.97}$

(iii) Compare the long-term behaviour of the two models. **[2]**

iii The first model suggests that the population will continue to increase with no limit

The second model suggests the population will tend towards a limit of $\frac{3988}{4.97} = 802$ as $t \to \infty$

END OF QUESTION PAPER